A MATHEMATICAL MODEL OF THE GENERATOR/STARTER FOR AUTOMOTIVE VEHICLES

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Abstract

In the paper has been formulated mathematical models of the not only AC commutatorless electromagnetically excited generator/starter but also brushless AC-DC/DC-AC commutator electromagnetically excited generator/starter, both based on the double electrical machine, analogous to the mathematical model of the brushed AC-DC commutator electromagnetically excited generator with a rotating AC-DC mechanoelectrical commutator taking into consideration equations of unholonomic constraints of the AC-DC commutator, a set of the second-order Euler-Lagrange differential equations of dynamics in a matrix notation for the AC commutatorless electromagnetically-excited generator can be written. After taking into account the equations of unholonomic constraints of the power diode and/or MOSFET electronic commutators in the differential equations of dynamics one can be obtained differential equations of dynamics, establishing of a mathematical model of AC-DC/DC-AC electronic-commutator electromagnetically-excited generator/starter with the diode and/or MOSFET AC-DC/DC-AC electronic commutator, acting as the double electrical machine's diode and/or MOSFET rectifier (during generating an electrical energy) and/or inverter (during not only cranking of the internal combustion engine, but also travelling of an automotive vehicles as the hybrid-electric vehicle.

Keywords: *brushless DC electrical machine, generator/starter, automotive generator*

1. Introduction

It recent years have been taken numerous research and development (R&D) works having as aim elaboration of an integrated electric energy supply and internal combustion engine (ICE) cranking system for modern automotive vehicles, with the possibility of elimination in this system up-to-now used brushed DC commutator electrical machines, namely: an electromechanical starter with the mechanoelectrical commutator and an automotive brushless DC generator.

It has significant influence on not only durability and reliability, but also on greater technical possibilities of up-to-now used solutions of the automotive ICE cranking and on-board electric energy supply systems.

Currently, advanced technology in power electronics (three-phase bridge systems produced as integrated circuits built-up on quick power diodes and fully controlled electrical valves – MOSFET, IGBT modules) make possible a replacement of two DC electrical machines by one electrical machine acting performing a few functions: not only an electromechanical starter and a mechanoelectrical generator but also ICE flywheel – „*three-in-one*“.

Besides, it is essential elimination of a belt for driving the generator as well as a clutch and pinion of the starter cooperating with the ICE crankshaft’s flywheel, which considerably increases durability and reliability of the system.

Conducted R&D works and emerging solutions of automotive generators/starters in giant automotive vehicle manufacturers has been concentrated on the concept based on brushless DC commutator electrical machines with the internal permanent magnet (IPM) magnetoelectrical excitation and the electronic commutator [2, 10].

Thus, in this paper has been presented a mathematical model of an automotive generator starter with the IPM magnetoelectrical excitation and the electronic commutator.

It is also well-known other solution of the generator/starter convinced by the author [6] based on using rotary transformer, which supply a brushless DC electrical machine's exciter.

2. Synthetic mathematical model of a discrete functional dynamical hypersystem

Formation of a mathematical model of the synthetic concrete functional dynamical hypersystem has been initiated by formation of the synthetic abstract dynamical hypersystem in systems approach and matrix notation [3, 4]. This approach based on principle of Hamilton-Ostrogradsky stationary action, permitting to take in consideration of a fulfilment work by all the generalised forces acting on an abstract dynamical system, on which is putting series conditions and constraints. Dynamic state of this dynamical hypersystem may be described by means of independent generalised coordinates, velocities as well as generalised extortion forces.

A synthetic mathematical model of the arbitrary abstract discrete dynamical system in systems approach and matrix notation has a form of Euler-Lagrange II order differential equation of dynamics [3, 4]:

$$\left[\frac{d}{dt}\left(\frac{\partial T^*}{\partial \|\dot{q}\|} \right) - \frac{\partial T^*}{\partial \|q\|} + \frac{\partial V}{\partial \|q\|} + \frac{d}{dt}\left(\frac{\partial T_F^*}{\partial \|\dot{q}\|} \right) - \|Q\|^T \right] \delta \|q\| = 0 \,, \tag{1}$$

where:

T^* - kinetic conservative co-energy of the arbitrary abstract dynamical hypersystem,

V - potential conservative energy of the arbitrary abstract dynamical hypersystem,

T_F^*- kinetic dissipative co-energy arbitrary abstract dynamical hypersystem,

$\|q\|$ - hypermatrix of the independent generalised coordinates,

$\|\dot{q}\|$ - hypermatrix derivative as regards time of the arbitrary abstract dynamical hypersystem,

$\|Q\|$ - hypermatrix of dissipative generalised extortion forces of the arbitrary abstract dynamical hypersystem.

In afore mentioned Euler-Lagrange II order differential equation of dynamics may be applied hypermatrix notation of generalised coordinates and velocities, respectively, as well as scalar functions of the kinetic conservative co-energy ***T**** and potential conservative energy ***V*** as quadratic forms.

The kinetic conservative co-energy ***T**** of the arbitrary abstract dynamical hypersystem with constant parameters may be rewritten in the form:

$$T^* \equiv \tfrac{1}{2} \|\dot{q}\|^T \|K(q, \dot{q})\| \|\dot{q}\|, \tag{2}$$

where:

$\|K(q,\dot{q})\| = \dfrac{\mathrm{d}\|p\|}{\mathrm{d}\|\dot{q}\|}$ - hypermatrix of the kinetic conservative coefficients (generalised inertia).

The potential conservative energy ***V*** of the arbitrary abstract dynamical hypersystem may be rewritten as the quadratic form, namely:

$$V \equiv \tfrac{1}{2} \|q\|^T \|P(q, \dot{q})\| \, \|q\|, \tag{3}$$

where:

$\{P\} = \dfrac{d\{f\}}{d\{q\}}$ - hypermatrix of the potential conservative coefficient

The kinetic dissipative co-energy $\boldsymbol{T_F}$* of the arbitrary abstract dynamical hypersystem may be defined as:

$$T_F^{\,*} = \int_0^t F_F \; dt = \int_0^t (F_V + F_C)\, dt \,, \tag{4}$$

where an expression under the integral (so-termed Rayleigh dissipation function) is a sum of the power losses on heat, existing during the viscous friction and the coulomb friction in the taken into consideration dynamical system.

Power losses on the viscose friction may be defined as the quadratic form, namely:

$$F_V \equiv \tfrac{1}{2}\left\|\dot{q}\right\|^T \left\|N(q, \dot{q})\right\| \left\|\dot{q}\right\| . \tag{5}$$

However, a second component of the dissipative function in Eq. (4) describes a quantity of the power loss on the coulomb friction (in mechanical dynamical system) as well as the power loss on heat in the electrical dynamical system (e.g., power losses in the electrical valves – diodes, transistors etc.).

$$F_C = \tfrac{1}{2}\left\|\dot{q}\right\|^T \left\|\Delta Q\right\| sign\left\|\dot{q}\right\| , \tag{6}$$

where:

$\|\Delta Q\|$ - hypermatrix of generalised dissipative forces' drops of the power losses on heat caused by the coulomb friction and voltage drops.

The dependence (4) after taking into account the expressions (5) and (6) may be rewritten as:

$$T_F^* \equiv \int_0^t \left(\tfrac{1}{2}\left\|\dot{q}\right\|^T \left\|N(q, \dot{q})\right\| \left\|\dot{q}\right\| + \tfrac{1}{2}\, sign\left\|\dot{q}\right\|^T \left\|\Delta Q\right\| \left\|\dot{q}\right\| \right) dt . \tag{7}$$

After taking into consideration dependencies (3), (5), (6) and (7) Eq. (1) may be rewritten in the form:

$$\left[\frac{d}{dt}\left(\left\|\dot{q}\right\|^T \left\|K(q, \dot{q}\right\| \right) - \frac{1}{2}\left\|\dot{q}\right\|^T \frac{\partial \left\|K(q, \dot{q})\right\|}{\partial \left\|q\right\|} \left\|\dot{q}\right\| + \left\|q\right\|^T \left\|P(q, \dot{q})\right\| + \left\|\dot{q}\right\|^T \left\|N(q, \dot{q}\right\| + sign\left\|\dot{q}\right\|^T \left\|\Delta Q\right\| - \left\|Q\right\|^T \right] \delta\left\|q\right\| = 0 . \tag{8}$$

The mathematical model represented by Eq. (8) is termed an auxiliary one, due to the fact that from it may be formulated mathematical models of dissimilar dynamical systems of the generators/starters realised on dissimilar electrical machines.

3. Mathematical model of the brushless double-supplied electrical machine with the electromagnetical excitation

A physical model of the brushless generator/starter with electronic commutators based on the electromagnetically exited double electrical machine is shown in Figure 1.

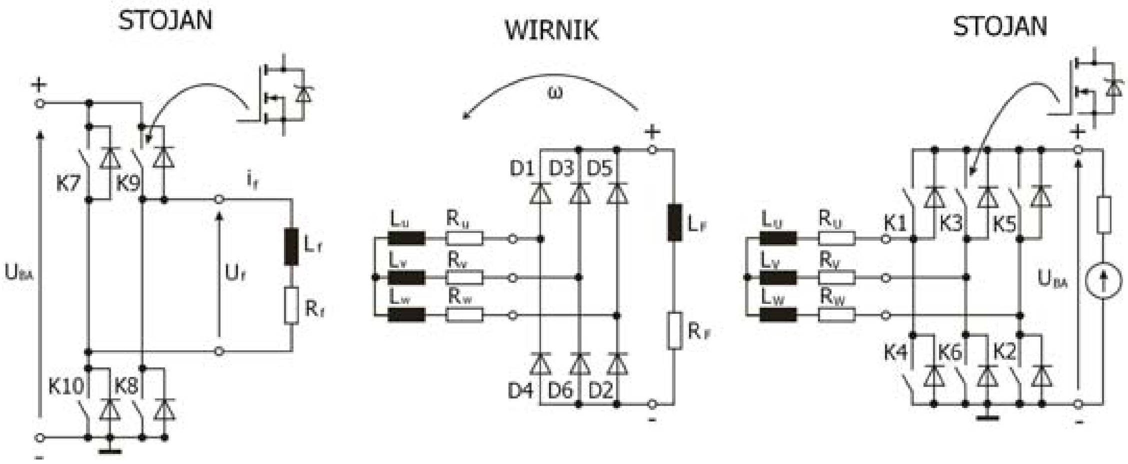

Fig. 1. The brushless generator/starter with electronic commutators based on the electromagnetically excited double electrical machine [9]

For simplifying mathematical description of these electrical machines in the following considerations have been taken into account only two electronic commutators, namely: the rotary electronic commutator on diodes D1 - D6 supplying an exciter of the main electrical machine as well the armature electronic commutator on six electronic keys K1 – K6 as well six diodes shunting those keys. Note: the auxiliary exciter's electronic commutator has been omitted during considerations of the field voltage U_f and field current i_f.

In a modified set of the second-order Euler-Lagrange differential equation of dynamics the regarded AC commutatorless generator/starter, that is, activity and state system on five activity and state hypo systems: a rotor mechanical hypo system, a rotor's electromagnetical excitation electrical hypo system of the main electrical machine, a stator's armature electrical system of the main electrical machine, a stator's electromagnetical excitation electrical hypo system of the auxiliary electrical machine and a rotor's armature electrical system of the auxiliary electrical machine hypo-systems, the particular low-level system vectors and matrices incorporating two low-level hypo system vectors and hypo matrices:

The hyper vector of the generalised coordinates and velocities:

$$\|q\| = \left\| [q^m]^T \ [q^e]^T \right\|^T, \quad \|\dot{q}\| = \left\| [\dot{q}^m]^T \ [\dot{q}^e]^T \right\|^T. \tag{9}$$

The hypo vector of the generalised forces and sources voltages:

$$\|Q\| = \left\| [Q^m]^T \ [Q^e]^T \right\|^T. \tag{10}$$

The diagonal matrix of the hypo matrix of generalised dissipative forces' drops of the power losses on heat caused by the coulomb friction and voltage drops:

$$\|\Delta Q\| = diag \, \| [\Delta Q^m] \ [\Delta Q^e] \|. \tag{11}$$

The hypermatrix of kinetic conservative coefficients of the mechanical system (moments of inertia) as well as electrical systems (self-and-mutual inductances):

$$\|K(q,\dot{q})\| = \begin{Vmatrix} [K^m(q,\dot{q})] & [0] \\ [0] & [K^e(q,\dot{q})] \end{Vmatrix}, \tag{12}$$

The hypermatrix of the potential conservative coefficients (generalised stiffnesses):

$$\|P(q,\dot{q})\| = \|0\| \tag{13}$$

due to fact that regarded AC commutatorless generator/starter there are no conservative elements saving potential energy, the shaft stiffness and capacitances has been omitted.

$$\left\| N(q,\dot{q}) \right\| = \left\| \begin{matrix} \left[N^{m}(q,\dot{q}) \right] & [0] \\ [0] & \left[N^{e}(q,\dot{q}) \right] \end{matrix} \right\|, \tag{14}$$

where index “*m*” apply to the mechanical system and index “*e*” apply to the electrical system.

Vectors of the generalized coordinates and velocities of the mechanical and electrical systems have been established by hypo vectors or single scalars:

$$\left\{ q^{m} \right\} = \theta, \quad \left\{ \dot{q}^{m} \right\} = \dot{\theta} = \omega, \quad \left\{ q^{m} \right\} = \left\{ \left[q^{f} \right]^{T} \left[q^{a} \right]^{T} \left[q^{F} \right]^{T} \left[q^{A} \right]^{T} \right\}^{T}, \tag{15}$$

$$\left[q^{f} \right] = q^{f}, \left[q^{F} \right] = q^{F}, \ \left[q^{a} \right] = \left[q^{u} \ q^{v} \ q^{w} \right]^{T}, \quad \left[q^{A} \right] = \left[q^{U} \ q^{V} \ q^{W} \right]^{T}, \tag{16}$$

where index “*a*” – apply to the armature of the auxiliary electrical machine, and index “*A*” – apply to the armature of the main electrical machine.

$$\left\{ \dot{q}^{e} \right\} = \left\{ \left[\dot{q}^{f} \right]^{T} \left[\dot{q}^{a} \right]^{T} \left[\dot{q}^{F} \right]^{T} \left[\dot{q}^{A} \right]^{T} \right\}^{T}, \ \left[\dot{q}^{f} \right] = i^{f} = i_{f}, \quad \left[\dot{q}^{F} \right] = i^{F} = i_{F}, \tag{17}$$

$$\left[\dot{q}^{a} \right] = \left[\dot{q}^{u} \ \dot{q}^{v} \ \dot{q}^{w} \right]^{T} = \left[i^{u} \ i^{v} \ i^{w} \right]^{T}, \quad \left[\dot{q}^{A} \right] = \left[\dot{q}^{U} \ \dot{q}^{V} \ \dot{q}^{W} \right]^{T} = \left[i^{U} \ i^{V} \ i^{W} \right]^{T}. \tag{18}$$

The three-phase armature winding of the auxiliary electrical machine has been signed by means of small letter indices *u, v, w*. However, the three-phase armature winding of the main electrical machine has been signed by means of capital letter indices *U, V, W*. Similarly, the field winding of the auxiliary machine has been signed by means small letter index „*f*”, but the field winding of the main electrical machine has been signed by index „*F*”.

The kinetic conservative co-energy as regards to the quadratic form from Eq. (2) of the regarded dynamical hypersystem may be rewritten in the form:

$$\begin{aligned} T^{*} = {} & \frac{1}{2} J(\theta)\omega^{2} + \frac{1}{2} L_{f} i_{f}^{\,2} + \frac{1}{2} L_{fu}(\theta) i_{f} i_{u} + \frac{1}{2} L_{fv}(\theta) i_{f} i_{u} + \frac{1}{2} L_{fw}(\theta) i_{f} i_{w} + \frac{1}{2} L_{uf}(\theta) i_{u} i_{f} + \\ & + \frac{1}{2} L_{vf}(\theta) i_{v} i_{f} + \frac{1}{2} L_{wf}(\theta) i_{w} i_{f} + \frac{1}{2} L_{u}(\theta) i_{u}^{\,2} + \frac{1}{2} L_{v}(\theta) i_{v}^{\,2} + \frac{1}{2} L_{w}(\theta) i_{w}^{\,2} + \frac{1}{2} L_{uv}(\theta) i_{u} i_{v} + \frac{1}{2} L_{vu}(\theta) i_{v} i_{u} + \\ & + \frac{1}{2} L_{uw}(\theta) i_{u} i_{w} + \frac{1}{2} L_{wu}(\theta) i_{w} i_{u} + \frac{1}{2} L_{u}(\theta) i_{u}^{\,2} + \frac{1}{2} L_{v}(\theta) i_{v}^{\,2} + \frac{1}{2} L_{w}(\theta) i_{w}^{\,2} + \frac{1}{2} L_{F}(\theta) i_{F}^{\,2} + \frac{1}{2} L_{FU}(\theta) i_{F} i_{U} + . \\ & + \frac{1}{2} L_{FV}(\theta) i_{F} i_{U} + \frac{1}{2} L_{FW}(\theta) i_{F} i_{W} + \frac{1}{2} L_{UF}(\theta) i_{U} i_{F} + \frac{1}{2} L_{VW}(\theta) i_{V} i_{F} + \frac{1}{2} L_{WF}(\theta) i_{W} i_{F} + \frac{1}{2} L_{U}(\theta) i_{U}^{\,2} + \\ & + \frac{1}{2} L_{V}(\theta) i_{V}^{\,2} + \frac{1}{2} L_{W}(\theta) i_{W}^{\,2} + \frac{1}{2} L_{UV}(\theta) i_{U} i_{V} + \frac{1}{2} L_{VU}(\theta) i_{V} i_{U} + \frac{1}{2} L_{UW}(\theta) i_{U} i_{W} + \frac{1}{2} L_{WU}(\theta) i_{W} i_{U} + \\ & + \frac{1}{2} L_{U}(\theta) i_{U}^{\,2} + \frac{1}{2} L_{V}(\theta) i_{V}^{\,2} + \frac{1}{2} L_{W}(\theta) i_{W}^{\,2} \end{aligned} \tag{19}$$

The hypo matrix of the kinetic coefficients kinetic conservative coefficients of the mechanical system (moments of inertia) as well as electrical systems (self-and-mutual inductances) contains with one conservative element – moment of inertia of the electrical machine as well as the ICE crankshaft mechanism and auxiliary equipment (water pomp, distribution shaft) reduced on a shaft of the electrical machine.

$$\left\{ K^{m}(q,\dot{q}) \right\} = \left\{ K^{m}(\theta,\omega) \right\} = J(\theta,\omega). \tag{20}$$

Subsequently, matrix of the kinetic conservative coefficients (self- and/or mutual-inductances) of the electrical dynamical system may be rewritten as follows:

$$\left\{K^m(q,\dot{q})\right\}=\begin{bmatrix} L^f & L^{fu}(\theta) & L^{fv}(\theta) & L^{fw}(\theta) & 0 & 0 & 0 & 0 \\ L^{uf}(\theta) & L^{u}(\theta) & L^{uv}(\theta) & L^{uw}(\theta) & 0 & 0 & 0 & 0 \\ L^{vf}(\theta) & L^{vu}(\theta) & L^{v}(\theta) & L^{vw}(\theta) & 0 & 0 & 0 & 0 \\ L^{wf}(\theta) & L^{wu}(\theta) & L^{wv}(\theta) & L^{w}(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & L^F & L^{FU}(\theta) & L^{FV}(\theta) & L^{FW}(\theta) \\ 0 & 0 & 0 & 0 & L^{UF}(\theta) & L^{U}(\theta) & L^{UV}(\theta) & L^{UW}(\theta) \\ 0 & 0 & 0 & 0 & L^{VF}(\theta) & L^{VU}(\theta) & L^{V}(\theta) & L^{VF}(\theta) \\ 0 & 0 & 0 & 0 & L^{WF}(\theta) & L^{WU}(\theta) & L^{WF}(\theta) & L^{W}(\theta) \end{bmatrix}. \tag{21}$$

The matrix of the kinetic dissipative coefficients of the mechanical dynamical system composes of only one dissipative element:

$$\left\{N^m(q,\dot{q})\right\}=D(\omega)=D, \tag{22}$$

that represents a viscose friction coefficient on the electrical machine and ICE shaft.

However, the matrix of the kinetic dissipative coefficients of the electrical dynamical systems has been rewritten in the form:

$$\left\{N^e(q,\dot{q})\right\}=diag\left\{\left[N^f\right]\left[N^a\right]\left[N^F\right]\left[N^A\right]\right\}=diag\left\{R^f R^u\ R^v\ R^w\ R^F\ R^U\ R^V\ R^W\right\} \tag{23}$$

and it may establish the diagonal matrix of electrical resistances.

The matrix of the generalised dissipative forces' drops, that is, the constant torque drop on the electrical machine and ICE shaft caused by the coulomb friction, existing in mechanical dynamical system and the constant voltage drops on the electrical valves, existing in electrical dynamical system:

$$\left\{\Delta Q^m\right\}=\Delta M, \tag{24}$$

$$\left\{\Delta Q^e\right\}=diag\ \left\{\left[\Delta Q^f\right]\left[\Delta Q^a\right]\left[\Delta Q^F\right]\left[\Delta Q^A\right]\right\}=diag\ \left\{\Delta U^f \Delta U^u\ \Delta U^v\ \Delta U^w\ \Delta U^F \Delta U^U\ \Delta U^V \Delta U^W\right\}. \tag{25}$$

The vector of the generalised extortion forces (input/output torques and voltages):

$$\left\{Q^m\right\}=M, \tag{26}$$

$$\left\{Q^e\right\}=\left\{(Q^f)^T(Q^a)^T(Q^F)^T(Q^A)^T\right\}^T=\left\{U^f U^u\ U^v\ U^w\ U^F\ U^U\ U^V\ U^W\right\}^T, \tag{27}$$

where: M – the input/output ICE torque.

A mathematical model of the mechanoelectrical discrete dynamical hypersystem is composed of the ICE and the double electrical machine on the common shaft may be rewritten in the form:

$$\left[\frac{d}{dt}\left(\left\|\begin{matrix} J(\theta) & 0 & 0 & 0 & 0 \\ 0 & L^f & [L^{fa}(\theta)] & 0 & 0 \\ 0 & [L^{af}(\theta)] & [L^a(\theta)] & 0 & 0 \\ 0 & 0 & 0 & L^F & [L^{FA}(\theta)] \\ 0 & 0 & 0 & [L^{AF}(\theta)] & [L^A(\theta)] \end{matrix}\right\|\left\|\begin{matrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{matrix}\right\|\right)+\right.$$

$$\left.+\left\|\begin{matrix} D_m & 0 & 0 & 0 & 0 \\ 0 & R^f & 0 & 0 & 0 \\ 0 & 0 & [R^a] & 0 & 0 \\ 0 & 0 & 0 & R^F & 0 \\ 0 & 0 & 0 & 0 & [R^A] \end{matrix}\right\|\left\|\begin{matrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{matrix}\right\|+sign\left\|\begin{matrix} \Delta M & 0 & 0 & 0 & 0 \\ 0 & \Delta u^f & 0 & 0 & 0 \\ 0 & 0 & [\Delta u^a] & 0 & 0 \\ 0 & 0 & 0 & \Delta u^F & 0 \\ 0 & 0 & 0 & 0 & [\Delta u^A] \end{matrix}\right\|\left\|\begin{matrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{matrix}\right\|-\left\|\begin{matrix} M_m \\ U^f \\ [U^a] \\ U^F \\ [U^A] \end{matrix}\right\|^T\right]=0. \tag{28}$$

The matrix of the electromechanical torque of the double electrical machine establishes a sum of the torque components on the ICE and electrical machine common shaft derived as well from the exciter (auxiliary electrical machine) as from the generator/starter (main electrical machine) may be rewritten in the quadratic form:

$$M_{em} = \frac{1}{2}\begin{Vmatrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{Vmatrix}^T \begin{Vmatrix} \frac{\partial J(\theta)}{\partial\theta} & 0 & 0 & 0 & 0 \\ 0 & 0 & [\frac{\partial L^{fa}(\theta)}{\partial\theta}] & 0 & 0 \\ 0 & [\frac{\partial L^{af}(\theta)}{\partial\theta}] & [\frac{\partial L^{a}(\theta)}{\partial\theta}] & 0 & 0 \\ 0 & 0 & 0 & 0 & [\frac{\partial L^{FA}(\theta)}{\partial\theta}] \\ 0 & 0 & 0 & [\frac{\partial L^{AF}(\theta)}{\partial\theta}] & [\frac{\partial L^{A}(\theta)}{\partial\theta}] \end{Vmatrix} \begin{Vmatrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{Vmatrix}, \qquad (29)$$

The electromagnetic torque on the ICE and electrical machine shaft generated by exciter is the braking torque, but the electromechanical torque of the generator/starter (main electrical machine) may change a sign of this torque in dependence of the mode of operation (generator/starter).

The mathematical model of the AC commutatorless generator/starter may be obtained by differentiating of Eq. (28), namely:

$$\omega \begin{Vmatrix} \frac{\partial J(\theta)}{\partial\theta} & 0 & 0 & 0 & 0 \\ 0 & 0 & [\frac{\partial L^{fa}(\theta)}{\partial\theta}] & 0 & 0 \\ 0 & [\frac{\partial L^{af}(\theta)}{\partial\theta}] & [\frac{\partial L^{a}(\theta)}{\partial\theta}] & 0 & 0 \\ 0 & 0 & 0 & 0 & [\frac{\partial L^{FA}(\theta)}{\partial\theta}] \\ 0 & 0 & 0 & [\frac{\partial L^{AF}(\theta)}{\partial\theta}] & [\frac{\partial L^{A}(\theta)}{\partial\theta}] \end{Vmatrix} \begin{Vmatrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{Vmatrix} + \begin{Vmatrix} L^f & [L^{fa}(\theta)] & 0 & 0 \\ [L^{af}(\theta)] & [L^{a}(\theta)] & 0 & 0 \\ 0 & 0 & L^F & [L^{FA}(\theta)] \\ 0 & 0 & [L^{AF}(\theta)] & [L^{A}(\theta)] \end{Vmatrix}$$

$$\frac{d}{dt}\begin{Vmatrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{Vmatrix} + \begin{Vmatrix} D_m & 0 & 0 & 0 & 0 \\ 0 & R^f & 0 & 0 & 0 \\ 0 & 0 & [R^a] & 0 & 0 \\ 0 & 0 & 0 & R^F & 0 \\ 0 & 0 & 0 & 0 & [R^A] \end{Vmatrix}\begin{Vmatrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{Vmatrix} + sign\begin{Vmatrix} \Delta M & 0 & 0 & 0 & 0 \\ 0 & \Delta u^f & 0 & 0 & 0 \\ 0 & 0 & [\Delta u^a] & 0 & 0 \\ 0 & 0 & 0 & \Delta u^F & 0 \\ 0 & 0 & 0 & 0 & [\Delta u^A] \end{Vmatrix}\begin{Vmatrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{Vmatrix} = \begin{Vmatrix} M_m \\ U^f \\ [U^a] \\ U^F \\ [U^A] \end{Vmatrix}. \qquad (30)$$

All afore-conducted considerations have been concerned of the mathematical model of the AC commutatorless electrical machine that establishes the mechanoelectrical discrete dynamical hypersystem with the holonomic constraints for which constraint equations may be integrated and without any restriction may be applied Euler-Lagrange differential equations of dynamics (1).

4. A mathematical model of the uncontrolled and controlled electronic commutator

A physical model of the generalised electronic commutator acting in three-phase bridge system, consisting of primary (input) and secondary (output) conducting electrical current collectors is shown in Figures 2.

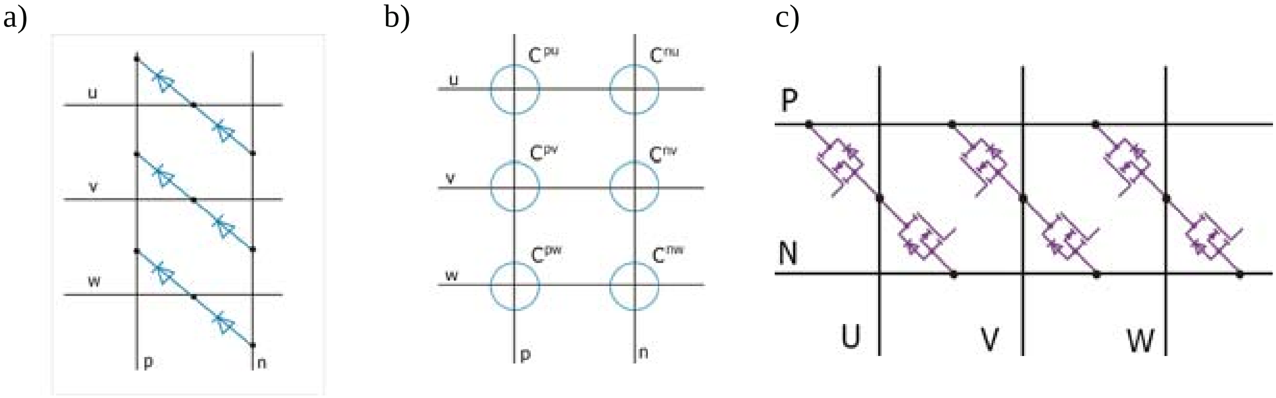

Fig. 2. The physical model of electronic commutators, acting as (a) an uncontrolled rectifier in the three-phase bridge system, (b) a matrix converter (matrixer) with the electrical valves C^{kl} as well (c) a controlled rectifier/inverter in the three-phase bridge system

In this model the primary (input) collectors have been signed with the index – *p*, but the secondary (output) collectors – with the index – *S*.

In a regarded case, the primary collectors are connected with secondary collector by means of the power diodes and/or MOSFETs or the other electrical valves (Fig. 1).

A physical model of the electronic commutator for the automotive brushless generator/starter (Fig. 2) as an uncontrolled and/or controlled rectifier with three primary (input) collectors and two secondary (output) collectors may be illustrated taking into account phase voltages of the generator/starter three-phase armature winding applied to the primary (input) collectors *u*, *v*, *w* of the armature electronic commutator in a form of the hypo matrix of the armature phase voltages:

$$[U^p] = [U_u \quad U_v \quad U_w]^T, \tag{31}$$

as primary (input) electrical quantities, and a rectified voltage applied to a secondary (output) collector *p* and *n* of the armature electronic commutator in a form of the hypo matrix of the rectified armature voltages:

$$[U^s] = [U^p \quad U^n]^T, \tag{32}$$

that may be rewritten as the secondary (output) electrical quantities.

Thus, a mathematical model of the electronic commutator acting as the voltage rectifier depending on an input from output in systems approach of dynamics and matrix notation may be rewritten in the form:

$$[U_S] = [C_{Sp}][U^p], \tag{33}$$

or in more detail:

$$\begin{bmatrix} U^p \\ U^n \end{bmatrix} = \begin{bmatrix} C^{pu} & C^{pv} & C^{pw} \\ C^{nu} & C^{nv} & C^{nw} \end{bmatrix} \begin{bmatrix} U^u \\ U^v \\ U^w \end{bmatrix}, \tag{34}$$

where elements of the commutating matrix C^{kl} are voltage coefficients of the commutating nodes (points of junction), respectively, nodes *'Kl'*, which for the ideal electrical valves may be represented in the following form:

$$C^{kl} = \begin{cases} \pm 1 \rightarrow i^k \ \ for\ (\theta - \alpha) \\ \ 0 \rightarrow i^k \ \ for\ (\theta - \alpha) \\ -1 \rightarrow i^k \ \ for\ (\theta - \alpha) \end{cases} . \tag{35}$$

In a regarded operation case of the electronic commutator voltage coefficients of the commutating nodes C^{kl} may be simulated by means respective physical macro-models of the real electrical valves (e.g., in the simulation program PSPICE).

The vector of the primary currents for three primary (input) collectors of the electronic commutator for the AC three-phase armature winding of the of the exciter (auxiliary electrical machine) may be presented as:

$$\left[i^p\right] = \left[i^u \ i^v \ i^w\right]^T , \tag{36}$$

however, the vector of the secondary currents for two secondary (output) collectors of the electronic commutator the DC two secondary (output) collectors of the above mentioned electronic commutator may be given as:

$$\left[i_S\right] = \left[i_p \quad i_n\right]^T . \tag{37}$$

in this case $i_p = - i_n = i_f$ and $i_P = - i_N = i_F$ (Fig.1).

Thus, the mathematical model of the electrical discrete dynamical hypo system of the exciter‘s electronic commutator of the automotive generator/starter acting as the current rectifier depending on an output from input may be represented in the following forms:

$$\left[i^p\right] = \left[C^{pS}\right]\left[i_S\right] , \tag{38}$$

$$\begin{bmatrix} i^u \\ i^v \\ i^w \end{bmatrix} = \begin{bmatrix} C^{up} & C^{un} \\ C^{vp} & C^{vn} \\ C^{wp} & C^{wn} \end{bmatrix} \begin{bmatrix} i_p \\ i_n \end{bmatrix} . \tag{39}$$

Comparing Eq. (39) with Eq. (34) the vector of the secondary (output) voltages as a function of the vector of the primary (input) voltages may be represented as follows:

$$\left[U_S\right] = \left[C^{pS}\right]^T \left[U^p\right], \tag{40}$$

where current coefficients C^{kl} of the commutating nodes '*kl*', of the commutating matrix [C^{pS}] for ideal electrical valves may be expressed as follows:

$$C_a^{\ kl} = C_a^{\ kl}[\max\left(U_u, U_v, U_w\right), \min\left(U_u, U_v, U_w\right)] , \tag{41}$$

where elements of the commutating matrix [C_a] of the exciter electronic commutator are acting as the rotary uncontrolled rectifier (Fig. 1) are functions of the armature phase voltages.

The armature electronic commutator of the generator/starter main electrical machine is acting with the commutating matrix [C_A] as well the uncontrolled rectifier (during action as an onboard generator) as fully-controlled inverter (during action as starter when cranking ICE or during travelling of the automotive vehicles as the hybrid-electric vehicle). It is possible due to the fact that in this electronic commutator are controlled bipolar electrical valves.

During action as a generator the armature electronic commutator of the generator/starter main electrical machine is acting similarly as a rotary electronic commutator (uncontrolled rectifier) of the generator/starter auxiliary electrical machine. For this reason its commutating matrix may be also defined by Eq. (41).

During action as a starter motor the armature electronic commutator of the generator/starter main electrical machine is acting as a fully controlled inverter. During motoring, it may be controlled in different modes. Thus, in dependence on the adapted mode of the electrical valves control of the inverter the elements of the commutating matrix $[C_A]$ may be defined as follows:

$$C_A{}^{Kl} = C_A{}^{Kl}\left[(\theta-\alpha)+(n-1)\frac{2\pi}{m}\right], \qquad (42)$$

$$K = U, V, W, \quad l = p, n; \quad n = 1, 2, 3; \quad m = 3.$$

where:

- α – control angular displacement, adequate to the *‚triggering–on‘* angle of the electronic commutator electrical valves,
- θ – angular displacement of the ICE and the electrical machine shaft, measured by means of the encoder or resolver.

5. A mathematical model of the brushless DC generator/starter

A commutating matrix $[C_a]$ for the exciter electronic commutator of the generator/starter auxiliary electrical machine that is acting as a rotary uncontrolled rectifier:

$$[C_a] = \begin{bmatrix} C^{up} & C^{un} \\ C^{vp} & C^{vn} \\ C^{wp} & C^{wn} \end{bmatrix}^T, \qquad (43)$$

and a commutating matrix $[C_A]$ for the armature electronic commutator of the generator/starter main electrical machine that is acting as well an uncontrolled rectifier during generating as a fully controlled inverter for motoring

$$[C_A] = \begin{bmatrix} C^{UP} & C^{UN} \\ C^{VP} & C^{VN} \\ C^{WP} & C^{WN} \end{bmatrix}^T. \qquad (44)$$

The hypermatrix of the generalised velocities of the mechanoelectrical discrete dynamical hypersystem consisting of the brushless AC-DC/DC-AC commutator electromagnetically-excited generator/starter may be defined in the form:

$$\|\dot{q}_S\| = \begin{Vmatrix} \omega \\ i_f \\ [i_a] \\ i_F \\ [i_A] \end{Vmatrix} = \begin{Vmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & [C_a] & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & [C_A] \end{Vmatrix} \begin{Vmatrix} \omega \\ i^f \\ [i^a] \\ i^F \\ [i^A] \end{Vmatrix}, \qquad (45)$$

where:

- $[i_a]$ – the vector of the armature current of the brushless AC-DC/DC-AC commutator electromagnetically-excited generator/starter auxiliary electrical machine,
- $[i_A]$ – the vector of the armature current of the brushless AC-DC/DC-AC commutator electromagnetically-excited generator/starter main electrical machine.

$$\left[U_a\right]=\left[R_a\right]\left[i_a\right]+\left[L_a\right]\frac{d}{dt}\left[i_a\right]+\omega\left[L^*{}_a\right]\left[i_a\right]+\left[\Delta U_a\right]sign\left[i_a\right], \qquad (46)$$

where:

$$\left[L^*{}_a\right]=\left[C_a{}^{kl}\right]^T\frac{d}{d\theta}\left[L^a\right]\left[C_a{}^{kl}\right], \quad \left[\Delta U_a\right]=\left[C_a{}^{kl}\right]^T\left[\Delta U^a\right], \quad \left[U_a\right]=\left[C_a{}^{kl}\right]^T\left[U^a\right],$$

$$\left[i_a\right]=\left[C_a{}^{kl}\right]^{-1}\left[i^a\right], \quad \left[R_a\right]=\left[C_a{}^{kl}\right]^T\left[R^a\right]\left[C_a{}^{kl}\right], \quad \left[L_a\right]=\left[C_a{}^{kl}\right]^T\left[L^a\right]\left[C_a{}^{kl}\right]. \qquad (47)$$

Similarly, for

$$\left[U_A\right]=\left[R_A\right]\left[i_A\right]+\left[L_A\right]\frac{d}{dt}\left[i_A\right]+\omega\left[L^*{}_A\right]\left[i_A\right]+\left[\Delta U_A\right]sign\left[i_A\right], \qquad (48)$$

where: $\left[L^*{}_A\right]=\left[C_A{}^{Kl}\right]^T\frac{d}{d\theta}\left[L^A\right]\left[C_A{}^{Kl}\right], \quad \left[\Delta U_A\right]=\left[C_A{}^{Kl}\right]^T\left[\Delta U^A\right]. \quad \left[U_A\right]=\left[C_A{}^{Kl}\right]^T\left[U^A\right]$, (49)

$$\left[i_A\right]=\left[C_A{}^{Kl}\right]^{-1}\left[i^A\right], \quad \left[R_A\right]=\left[C_A{}^{Kl}\right]^T\left[R^A\right]\left[C_A{}^{Kl}\right], \quad \left[\Delta U_A\right]=\left[C_A{}^{Kl}\right]^T\left[\Delta U^A\right]. \qquad (50)$$

6. Conclusions

It has been shown, that mathematical model in the paper take into account some phenomena and parameters usually omitted in the approximated analytical studies of dynamics of automotive mechanoelectrical discrete dynamical hyper systems encountered in literature do not raise procedural difficulties with the representation of algorithm in programming language and allow for an acquisition of sufficient results accuracy of the computer analytical studies of dynamics.

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